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Temperature dependence of critical currents of two-gap superconductors

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Abstract. In this paper, we consider the two-gap Ginzburg-Landau (G-L) free energy functional including the interactions, and obtain a very simple and explicit formula which presents the relation between the critical current density and temperature. The result shows that the temperature dependence of critical current density of a two-gap superconductor is of the form $J_c \propto (T_c{}^* - T)^{1/2}$ at low temperatures and $J_c \propto (T_c^* - T)^{3/2}$ near the critical temperature. Our critical current density expression is in accord with the previous theoretical work and experimental data.

PACS. 74.20.De Phenomenological theories (two-fluid, Ginzburg-Landau, etc.) – 74.25.Sv Critical currents – 74.70.Ad Metals; alloys and binary compounds (including A15, MgB2, etc.) – 74.70.Dd Ternary, quaternary, and multinary compounds (including Chevrel phases, borocarbides, etc.)

1 Introduction

Many types of superconductor are under production for technological applications, and therefore their critical temperatures, fields and current densities have become important. The recently discovered superconductors rare-earth transition-metal borocarbides RNi_2B_2C, niobium diselenide $NbSe_2$ and the intermetallic compound magnesium diboride MgB_2 have attracted the interest of many groups. Borocarbide compounds with R = Y, Lu have fairly high critical temperatures, T_c, of about 15 K. Transition from normal phase to superconducting phase occurs in $NbSe_2$ at 7 K, and in MgB_2 at about 40 K, which is the highest transition temperature among the intermetallic compounds. It is also shown that their structure is made from an alternating sequence of lattice planes. As a result of their crystalline structure, these materials are expected to exhibit two-dimensional superconductivity.

Besides their unexpectedly high T_c, their high critical fields and the existence of two energy gap in the superconducting state attracts interest for theoretical and experimental work. In two-gap superconductors a Fermi surface passes through two bands, thus one must take two superconductive condensates into account for theoretical investigations. Calculations of the band structure and phonon spectrum predict the existence of two energy gap parameters for MgB_2 [1,2]. This two-gap characteristic of MgB_2 has become clearly evident in the recently performed tunnelling measurements [3,4] and specific heat measurements [5]. Quantum oscillation measurements of borocarbides $Lu(Y)Ni_2B_2C$ give clear evidence for multi-gap character in the normal conducting state [6]. Based on angle resolved photoemission measurements, it has also been proposed that multi-gap superconductivity occurs in $NbSe_2$ [7].

Both magnesium diboride and rare-earth borocarbides are also of interest due to their unusual magnetic properties. Some groups have shown that in contrast to conventional superconductors, the upper critical field of both these materials has positive curvature near the transition point [8–11]. To understand the nature of the unusual behavior at a microscopic level, a two-band Eliashberg model of superconductivity was first proposed by Shulga et al. [12] for $LuNi_2B_2C$ and YNi_2B_2C and recently, for MgB_2 [13]. Here it is necessary to remark that generalization of the BCS theory to the multi-band model was first suggested in references [14,15]. In reference [16] a two-band model was proposed in the strong coupling regime for describing the properties of high temperature cuprate superconductors. The two-band G-L model was first applied for the calculation of the upper critical field H_{c2} by Doh et al. [17]. More recently, similar two band theory was applied to find the temperature dependence of upper [18], lower [19] and thermodynamic [20] critical fields.

Critical current density J_c is one of important characteristics of the superconducting state. As mentioned in [21], high critical current densities have been observed

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in MgB_2 bulk samples regardless of the degree of grain alignment. This is of advantage for making wires, films or tapes with no degradation of J_c in contrast to cuprate superconductors. Based on G-L theory, the temperature dependence of the critical current in MgB_2 was investigated by Kunchur [22,23]. To explain critical current with varying temperature, there are also theoretical models especially for layered superconductors, such as the effective mass model [24], pinning model [25] and the two-dimensional model [26]. Dhalle [27] and Kim [28] reported the transport measurements in high magnetic fields.

G-L free energy functional has been used in order to describe the phase transitions. In its simplest classical form, the G-L functional describes the thermodynamic and static phenomena of superconductivity. To describe the dynamical properties, such as pairing fluctuations, one needs to take time dependent fields into account. Ginzburg and Landau proposed that a superconducting phase can be described by the thermodynamic variables temperature, density and a complex order parameter [29]. Further, Gorkov showed this that order parameter is directly proportional to the energy gap parameter [30].

The purpose of this paper is to derive the temperature dependence of critical current in the framework of two-gap G-L theory. First, the well-known Ginzburg-Landau equation is obtained for each band in the existence of interband interactions. The velocity of each Cooper pair is obtained from the first Ginzburg-Landau equation and substituted into the supercurrent density. Maximal point of the supercurrent density with respect to current carrier velocity is obtained as a critical current density.

2 Two-gap G-L free energy functional

According to the Landau theory of second order phase transitions, in the absence of a magnetic field, the free energy density of a two band superconductor has the following form:

$$F_{s0} = F_{n0} + \sum_{i=1}^{2}\left(\alpha_i\left|\Psi_i\right|^2 + \frac{\beta_i}{2}\left|\Psi_i\right|^4 + \frac{\hbar^2}{4m_i}\left|\nabla\Psi_i\right|^2\right) \quad (1)$$

where $\alpha_i = \gamma_i(T - T_{ci})$ and β_i is a positive constant, the effective mass of each Cooper pair is $2m_i$, F_{s0} is the free energy density of the superconducting state and F_{n0} for a normal state. The last term describes an additional energy which is due to the non-uniform state of a superconductor. We will consider the order parameters to be characterized by different effective masses, different densities and different critical temperatures.

If the magnetic field is zero inside the specimen, we then take $|\Psi_i|$ as constant. Minimization of F_{s0} with respect to Ψ_i gives

$$\left|\Psi_i\right|^2 = \frac{\gamma_i}{\beta_i}\left(T_{ci} - T\right). \quad (2)$$

The local density of superelectrons of the i th band is given by $n_i = 2|\Psi_i|^2$. Physically this means that all the superelectrons of the i th band are described by the same wave function Ψ_i.

If a magnetic field $\boldsymbol{H}$ is applied to the sample, the gradient term is modified as

$$\nabla^{'} = \nabla - \frac{2\pi i A}{\Phi_0}$$

where Φ_0 is the magnetic flux quantum and $\mathrm{rot}\boldsymbol{A}=\boldsymbol{H}$. The existence of this magnetic field $\boldsymbol{H}$ leads to an increase of free energy density by $H^2/8\pi$ per unit volume of superconductor.

Furthermore, the presence of two order parameters and their coupling cause an additional interaction energy term as follows.

$$F_{int} = \varepsilon\left(\Psi_1^*\Psi_2 + \Psi_1\Psi_2^*\right) + \varepsilon_1\hbar^2\left(\left(\nabla^{'}\Psi_1\right)^*\left(\nabla^{'}\Psi_2\right) + \left(\nabla^{'}\Psi_1\right)\left(\nabla^{'}\Psi_2\right)^*\right). \quad (3)$$

Here, the first term describes interband coupling of order parameters and ε is a characteristic of the interband coupling strength. The same interaction term was invoked to investigate the pressure effect on a two-gap superconductor MgB_2 [31] and the existence of a soliton in two-gap superconductors [32]. The second term $\varepsilon_1$describes interband mixing of gradients of two order parameters, which was given before for applications of the two band G-L approach to MgB_2 and borocarbides in references [18–20]. In order to generalize the London free energy for lattice vortices, by using similar interband mixing of gradient terms, Affleck et al. [33] considered the *d*-wave superconductors in zero magnetic field. A similar two-condensate G-L approach, Faddeev model, was also used in high energy physics for the investigation of hidden symmetries, background charge and knotted solitons [34]. The G-L free energy functional may also contain higher order terms of order parameters and differential operators.

Thus, the free energy density of a two band superconductor is obtained as

$$F_{sH} = F_{n0} + \sum_{i=1}^{2}\left(\alpha_i(T)\left|\Psi_i\right|^2 + \frac{\beta_i\left|\Psi_i\right|^4}{2} + \frac{\hbar^2}{4m_i}\left|\nabla^{'}\Psi_i\right|^2\right) + \varepsilon\left(\Psi_1^*\Psi_2 + \Psi_1\Psi_2^*\right) + \varepsilon_1\hbar^2\left(\left(\nabla^{'}\Psi_1\right)^*\left(\nabla^{'}\Psi_2\right) + \left(\nabla^{'}\Psi_1\right)\left(\nabla^{'}\Psi_2\right)^*\right) + \frac{H^2}{8\pi}. \quad (4)$$

3 Critical current density

Critical current density J_c is another characteristic of the superconducting state. As the supercurrent density increases, the supercurrent velocity starts to increase. But, this increase causes a decrease in current carrier density. Electron pairs start to break up at this point and after this there are not enough current carriers to maintain a higher current. Thus this maximum value of the supercurrent is called the critical supercurrent.

For a two band superconductor, if the strength of the superconducting state has small variation, we can introduce the order parameters $\Psi_n(r)$ by $|\Psi_n|\exp(i\theta_n)$, where amplitude $|\Psi_n|$ is constant $(n = 1, 2)$.

Hence, the free energy includes two variables n_i and $\boldsymbol{A}$. The behavior of a superconductor is determined by the condition that free energy density is a minimum with respect to these variables. By writing the order parameter as given above and by choosing the London gauge $\boldsymbol{\nabla}\cdot\boldsymbol{A} = 0$, we can minimize the free energy density with respect to the order parameter. This calculation gives us

$$\frac{m_i V_i^2}{4} + \alpha_i + \beta_i \frac{n_i}{2} + \left(\frac{n_j}{n_i}\right)^{\frac{1}{2}} (\varepsilon + \varepsilon_1 m_i m_j V_i V_j)\cos(\theta_i - \theta_j) = 0 \quad (i \neq j = 1, 2). \quad (5)$$

This is the first of the two-band G- L differential equation where the current carrier velocity is

$$\vec{V}_i = \frac{\hbar}{m_i}\vec{\nabla}\theta_i - \frac{2e\vec{A}}{m_i c} \quad (6)$$

and $\theta_i(r)$ is the position dependent phase of order parameter of each band.

Existence of a phase difference between two order parameters plays an important role in creating the superconducting state. Our free energy functional (4) and G-L equation (5) contain this phase difference, in a cosine term, due to interband interaction effects. In the case of nonzero ε the phase difference of order parameters at equilibrium can be given as

$$\theta_1 - \theta_2 = 0, \quad \text{if} \quad \varepsilon < 0 \quad (7a)$$

$$\theta_1 - \theta_2 = \pi, \quad \text{if} \quad \varepsilon > 0. \quad (7b)$$

As mentioned earlier in references [32,34,37], minimization of the free energy functional (4) requires the above conditions

To find the supercurrent density, we first minimize the free energy density (4) with respect to the variation of the vector potential $\boldsymbol{A}$. Then by using equation $\text{rot}\boldsymbol{H} = 4\pi\boldsymbol{J}/c$ we find

$$J_s = 2e\left(\frac{n_i}{4}V_i + \frac{n_j}{4}V_j + \varepsilon_1 (n_i n_j)^{\frac{1}{2}} (m_i V_i + m_j V_j)\right).$$

Equation (5) relates the Cooper pair velocity V_i to V_j. If we rewrite the equation considering the first G-L equations, the supercurrent density for two band superconducting materials becomes

$$J_s = 2e\left(V_i\eta_i + \eta_j\left(\frac{n_i m_i}{n_j m_j}V_i^2 + U(T)\right)^{\frac{1}{2}}\right) \quad (8)$$

where η_1 and η_2 are the effective densities of superelectrons

$$\eta_i = \frac{n_i}{4} + \varepsilon_1 m_i (n_i n_j)^{1/2} \quad (9)$$

and

$$U(T) = \frac{2}{m_j}\Big[2(C\alpha_i - \alpha_j) + Cn_i\beta_j - \beta_i n_j\Big]. \quad (10)$$

Here it is supposed that $n_i = Cn_j$. Supercurrent density takes the maximum value when supercurrent velocity reaches a critical value V_c, which can be obtained from the condition $\delta J_s/\delta V_i = 0$. Therefore, the critical velocity

$$V_c^2 = \frac{U(T)}{\left(\left(\frac{\eta_j}{\eta_i}\frac{n_i m_i}{n_j m_j}\right)^2 - \frac{n_i m_i}{n_j m_j}\right)}. \quad (11)$$

and the critical current density expression then becomes

$$J_c(T) = 2e\eta_i\sigma\,(U(T))^{\frac{1}{2}}. \quad (12)$$

where $2e$ is the charge of each Cooper pair and the parameters are

$$a = \frac{\eta_j}{\eta_i}, \quad b = \frac{n_j m_j}{n_i m_i}, \quad \sigma = \frac{a^2 + b}{(a^2 - b)^{\frac{1}{2}}}. \quad (13)$$

4 Results and discussion

In the present work, we obtain the critical current density from the isotropic two-gap Ginzburg-Landau theory and give the J_c expression in a relatively simple form in equation (12). The term $U(T)$ is effective at both low and high temperatures. Here it can be assumed that parameters a, b and σ have no temperature dependence. This can be seen from the similarity of the temperature dependence of the order parameters. Many experimental results show that the variation of gaps with temperature is rather small at low temperatures. Therefore, this allows us to define the densities of Cooper pairs as $n_i = \gamma_i^* T_{ci}/\beta_i$. After substituting this into equation (12), our theoretical expression, at low temperatures, becomes

$$J_c(T) = J_c(0)\,(T_c^* - T)^{\frac{1}{2}} \quad (14)$$

with

$$T_c^* = \frac{T_{cj}(\gamma_j - \gamma_j^*) - CT_{ci}(\gamma_i - \gamma_i^*)}{(2\gamma_j - 2C\gamma_i)}. \quad (15)$$

Equation (14) is in accord with the experimental data for two band superconductors MgB_2 [22,27,28] and borocarbides [35]. This result differs from single-gap G-L theory.

As the temperature is raised, both order parameters becomes small and their ratio constant [39,40]. However, superconducting gaps have strong temperature dependence near the critical temperature. Thus, the temperature dependence of effective density of superconducting electrons can be given approximately as [36]

$$\eta_i(T) \approx \eta_i(0)(T_c^* - T) \quad T \approx T_c^*. \quad (16)$$

Thus, in addition to temperature dependence of $U(T)$ being proportional to $(T_c^* - T)^{1/2}$ we may take temperature dependence of effective current density into account as above. So the expression for critical current density can be given for high temperatures as

$$J_c(T) \approx J_c(0)\,(T_c^* - T)^{\frac{3}{2}}. \quad (17)$$

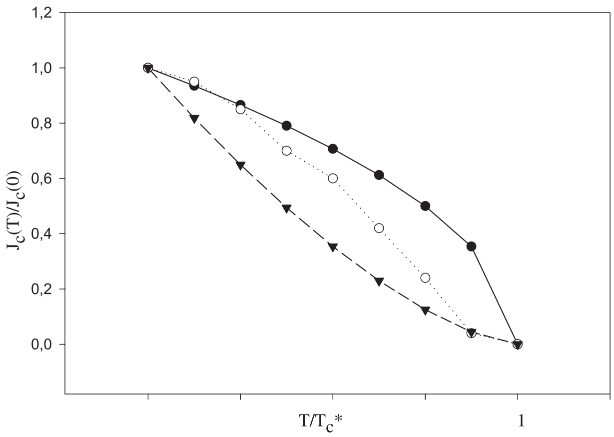

Fig. 1. $J_c(T)/J_c(0)$ versus reduced temperature. Full circles show two band G-L calculations, open circles show experimental data [22] and triangles show single band G-L calculations.

In Figure 1 We compare the results of single-gap and two-gap G-L theories with experimental data. Single-gap theory is only in agreement with experimental data near the transition point. Two-gap theory gives good description of variation of critical current density with temperature for both low and high temperatures. As shown before the parameters of MgB_2, $T_{c1} = 20$ K and $T_{c2} = 10$ K, were reported before in references [18–20,37,38]. These parameters give a good description of temperature dependencies of the upper critical field, lower critical field, thermodynamical critical field, coherence and penetration length and upper critical field anisotropy parameter. As seen from Figure 1 at low temperatures the experimental data of Kunchur [22] can be well approximated by two band G-L theory. This calculation also seems to be attractive for application to another class of two-band superconductor-nonmagnetic borocarbides $Y(Lu)Ni_2B_2C$. Another interesting related problem is the calculation of critical current density in the presence of an external magnetic field. A magnetic field dependent coefficient of two-band G-L theory was very recently used in reference [41].

In summary, in this paper we have calculated the temperature dependence of critical current density using two band G-L theory. It is shown that at low temperatures two-band G-L theory gives temperature dependence as $(T_c^* - T)^{1/2}$, which is in good agreement with experimental data. However at high temperatures, close to T_c, two band theory gives the behavior $(T_c^* - T)^{3/2}$.

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